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Operational modal analysis in the presence of harmonic excitation

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Abstract

Modal operational analysis methods are procedures to identify modal parameters of structures from the response to unknown random excitations existing on buildings and in machines during operation. In many practical cases, in addition to the random loads, harmonic excitations are also present due for instance to rotating components. If the frequency of the harmonic component of the input is close to an eigenfrequency of the system, operational modal analysis procedures fail to identify the modal parameters accurately. Therefore, we propose a modification of the least-square complex exponential identification procedure to include explicitly the harmonic component. In that way, the modal parameters can be identified properly. We illustrate the efficiency of the proposed approach on the example of a beam structure excited by multi-harmonic loads superposed on random excitation.

1. Introduction

Operational modal analysis (OMA) is a method, which allows a structure to be tested under operating conditions. At present, OMA is limited to the case when the excitation loads can be assimilated to stationary white-noise inputs. In practice however, many structures are vibrating due to harmonic excitation in addition to stationary white noise. Harmonic excitation can occur due to components like unbalanced rotors or fluctuating forces in electric actuators. Due to the presence of harmonic excitation, the modal identification procedures might lose their robustness and lead to inaccurate identified modal parameters.

A straightforward way of dealing with the harmonic response content in measured operational signals consists in regarding them as being the response of virtual modes having zero damping.

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So while doing modal parameter identification of the whole signal, additional frequencies associated to very low (theoretically zero) damping will be present and must then be spotted by the analyst.

In practice, due to the presence of harmonics close to the structural mode, identification methods usually give bad convergence of the parameters as the order of the identified model is increased. This problem often occurs when trying to identify parameters when the eigenspectrum is dense. When applying the polyreference least-square complex exponential (LSCE) method to the impulse-like response derived from the random excitation, stability of the frequency lines might converge to wrong modal parameters.

Special numerical filters can also be applied to filter-out harmonic components from the measured response. Unfortunately in practice, filters are not perfect and if the harmonic frequency is close to eigenfrequencies, the filtering will pollute the measured response so that the identified modal parameters are perturbed.

In Ref. [1], a procedure based on statistical properties of the outputs [2] was proposed in order to distinguish between harmonic response and narrowband stochastic response for output-only modal testing. The authors in Ref. [1] define an indicator to identify harmonics from the signal when the frequency domain decomposition is applied, but the example presented exhibits harmonic frequencies well separate from the eigenfrequencies.

In this work, we are considering cases where the harmonic frequencies are close to the eigenfrequencies. The single-reference LSCE method is considered as time-domain identification algorithm. Observing that the frequency of the harmonic response part can be easily evaluated either from the operational conditions or from the measured signal, we propose to explicitly include non-damped harmonic components in the identification procedure. In that way, we force the algorithm to identify the harmonic part of known frequency and zero damping.

In the next section, we briefly recall some theoretical background of an OMA method using LSCE identification. We outline the modification we propose in order to account for harmonic components in the response. Then in Section 3, we present a case study of a beam to illustrate the effectiveness of the procedure.

2. Operational modal analysis

2.1. Natural excitation technique and complex exponential identification

Assuming that a system is excited by a stationary white noise, it has been shown in the natural excitation technique (NExT) [3,4] that the correlation function $R_{ij}(t)$ between the response signals i and j at a time interval of t is similar to the response of the structure at i due to an impulse at j. Assuming damping to be small, this is expressed by the relation [4]

$$R_{ij}(t) = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} q_i(\tau)q_j(\tau - t)\,\mathrm{d}\tau = \sum_{r=1}^{N} \frac{\phi_{ri}A_{rj}}{m_r\omega_r^d}\,\mathrm{e}^{(-\zeta_r\omega_r^n t)}\sin(\omega_r^d t + \theta_r), \tag{1}$$

where ϕ_{ri} is the ith component of the eigenmode number r of the conservative system, A_{rj} is a constant associated to the jth response signal taken as reference, m_r is the rth modal mass, ζ_r and ω_r^n are, respectively, the rth modal damping ratio and non-damped eigenfrequency,

$\omega_r^d = \omega_r^n \sqrt{1-\zeta_r^2}$ and θ_r is the phase angle associated with the rth modal response. Hence, the correlation between signals is a superposition of decaying oscillations having damping and frequencies equal to the damping and frequencies of the structural mode.

As a consequence, modal parameter identification techniques like the LSCE method [5] can be used to extract the modal parameters from the correlation functions between measured responses to the noise input. In terms of the complex modes of the structure, the correlation function (1) can be written as (see for instance [6])

$$R_{ij}(k\Delta t) = \sum_{r=1}^{N} \psi_{ri} \mathrm{e}^{s_r k \Delta t} C_{rj} + \sum_{r=1}^{N} \psi_{ri}^{*} \mathrm{e}^{s_r^{*} k \Delta t} C_{rj}^{*}, \tag{2}$$

where $s_r = \omega_r \zeta_r + \mathrm{i}\omega_r \sqrt{(1-\zeta_r^2)}$ and where C_{rj} is a constant associated with the rth mode for the jth response signal, which is the reference signal. Δt is the sampling time step and the superscript * denotes the complex conjugate. Note that in conventional modal analysis, these constant multipliers are modal participation factors. Numbering all complex modes and eigenvalues in sequence, Eq. (2) can be written as

$$R_{ij}(k\Delta t) = \sum_{r=1}^{2N} C'_{rij} \mathrm{e}^{s_r k \Delta t}. \tag{3}$$

As s_r appears in complex conjugate forms in this expression, there exists a polynomial of order $2N$ (known as Prony's equation) of which $\mathrm{e}^{s_r \Delta t}$ are roots:

$$\beta_0 + \beta_1 V_r^1 + \beta_2 V_r^2 + \cdots + \beta_{2N-1} V_r^{2N-1} + V_r^{2N} = 0, \tag{4}$$

where $V_r = \mathrm{e}^{s_r \Delta t}$ and where $\beta_{2N} = 1$. $\{\boldsymbol{\beta}\}$ is the coefficient matrix of the polynomial. To determine the values of β_i, let us multiply the impulse response (3) for sample k by the coefficient β_k and sum up these values for $k = 0, \ldots, 2N$ [7–9]:

$$\sum_{k=0}^{2N} \beta_k R_{ij}(k\Delta t) = \sum_{k=0}^{2N} \left(\beta_k \sum_{r=1}^{2N} C'_{rij} V_r^k \right) = \sum_{r=1}^{2N} \left(C'_{rij} \sum_{k=0}^{2N} \beta_k V_r^k \right) = 0. \tag{5}$$

Hence, the coefficient $\{\boldsymbol{\beta}\}$ satisfy a linear equation whose coefficients are the impulse responses (or correlation functions) at $(2N+1)$ successive time samples. In order to determine those coefficients, relation (5) is written $2N$ times, starting at successive time samples. In other words, $2N$ Prony's equations are written to build up a linear system that determines the coefficients $\{\boldsymbol{\beta}\}$:

$$\beta_0 R_n + \beta_1 R_{n+1} + \cdots + \beta_{2N-1} R_{n+2N-1} = -R_{n+2N}, \quad n = 0, \ldots, 2N-1, \tag{6}$$

where, to simplify the notations, we use $R_k = R_{ij}(k\Delta t)$. From Eq. (6), we obtain $2N$ equations to determine $\{\boldsymbol{\beta}\}$. Note that more equations like (6) can be used to form an over-determined set of equations for $\{\boldsymbol{\beta}\}$. This might be useful in order to average out measurement noise for instance. In that case, n in the system of Eq. (6) is varied till $L \geqslant 2N-1$. The linear system (6) can be re-arranged and put in matrix form as

$$[\mathbf{R}]\{\boldsymbol{\beta}\} = -\{\mathbf{R}'\}. \tag{7}$$

[R] is an $(L \times 2N)$ matrix whose rows are the sequence of sampled impulse response. Once the coefficients $\{\boldsymbol{\beta}\}$ are computed by solving Eq. (7), the complex eigenvalues s_r are found by computing the roots of Prony's polynomial (4).

The procedure as explained so far uses a single correlation function. In order to improve the robustness of the method, one can write Eq. (7) for p correlation functions of several measured degrees of freedom with respect to a single response signal as reference. One then obtains the over-determined system:

$$\begin{bmatrix} [\mathbf{R}]_1 \\ [\mathbf{R}]_2 \\ \vdots \\ [\mathbf{R}]_p \end{bmatrix} \{\boldsymbol{\beta}\} = -\begin{Bmatrix} \{\mathbf{R}'\}_1 \\ \{\mathbf{R}'\}_2 \\ \vdots \\ \{\mathbf{R}'\}_p \end{Bmatrix}. \tag{8}$$

A solution in a least-square sense can be found for $\{\boldsymbol{\beta}\}$ from Eq. (8). This can be achieved for instance using a pseudo-inverse (see e.g., Ref. [10]). This technique is known as the single-reference Least-Square Complex Exponential (LSCE) method.

Another variant of the LSCE approach uses response function with respect to several references. Details can be found, e.g., in Ref. [8]. The polyreference LSCE is commonly considered as an efficient identification method and will be used here as reference method.

2.2. OMA in the presence of harmonic excitation

If the signal contains a forced harmonic part due to a harmonic excitation on top of the response to the noise excitation, the correlation between measured data will, in addition to the impulse response described earlier, contain a non-damped harmonic part. This extra component can be looked at as a non-damped virtual eigenmode of the system. Hence, the OMA procedure described above could be used even in the presence of harmonic excitation. However, explicitly including the harmonic in the identification procedure leads to a more efficient and robust approach as explained next.

When a harmonic excitation of known (or a priori identified) frequency ω is present, it can easily be shown that the correlation functions have the form (3) with two extra terms corresponding to eigenvalues $s_r = \pm i\omega$. There are thus two extra roots of Prony's polynomial, namely $V_r = e^{s_r \Delta t} = e^{\pm i\omega \Delta t}$.

Let us now explicitly express that $V_r = \cos(\omega_r \Delta t) - i \sin(\omega_r \Delta t)$ and $V_r = \cos(\omega_r \Delta t) + i \sin(\omega_r \Delta t)$ are roots of Eq. (6). Re-arranging the Prony's equations for these two eigenvalues, we find

$$\begin{bmatrix} 0 & \sin(\omega \Delta t) & \dots & \sin(\omega(2N-1)\Delta t) \\ 1 & \cos(\omega \Delta t) & \dots & \cos(\omega(2N-1)\Delta t) \end{bmatrix} \begin{Bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_{2N-1} \end{Bmatrix} = -\begin{Bmatrix} \sin(2N\omega \Delta t) \\ \cos(2N\omega \Delta t) \end{Bmatrix}. \tag{9}$$

These two independent linear relations for the coefficients $\{\boldsymbol{\beta}\}$ must be satisfied by Prony's coefficients to represent exactly the known harmonic components.

In general, we will assume there are m harmonic frequencies in the signal within the range of frequencies considered. Adding the linear relations (9) to the linear system (8), we get

$$\left[\begin{array}{ccc|ccc} R_0 & \dots & R_{2m-1} & R_{2m} & \dots & R_{2N-1} \\ \vdots & \mathrm{A} & \vdots & \vdots & \mathrm{C} & \vdots \\ R_{Lp-1} & \dots & R_{Lp+2m-2} & R_{Lp+2m-1} & \dots & R_{Lp+2N-2} \\ \hline 0 & \dots & \sin(\omega_1(2m-1)\Delta t) & \sin(\omega_1 2m\Delta t) & \dots & \sin(\omega_1(2N-1)\Delta t) \\ 1 & \dots & \cos(\omega_1(2m-1)\Delta t) & \cos(\omega_1 2m\Delta t) & \dots & \cos(\omega_1(2N-1)\Delta t) \\ \vdots & \mathrm{B} & \vdots & \vdots & \mathrm{D} & \vdots \\ 0 & \dots & \sin(\omega_m(2m-1)\Delta t) & \sin(\omega_m 2m\Delta t) & \dots & \sin(\omega_m(2N-1)\Delta t) \\ 1 & \dots & \cos(\omega_m(2m-1)\Delta t) & \cos(\omega_m 2m\Delta t) & \dots & \sin(\omega_m(2N-1)\Delta t) \end{array}\right] \left\{\begin{array}{c} \beta_0 \\ \beta_1 \\ \mathbf{b}_1 \\ \beta_{2m-2} \\ \beta_{2m-1} \\ \hline \beta_{2m} \\ \mathbf{b}_2 \\ \beta_{2N-1} \end{array}\right\}$$

$$= -\left\{\begin{array}{c} R_{2N} \\ \mathrm{E} \\ R_{Lp+2N-1} \\ \hline \sin(\omega_1 2N\Delta t) \\ \cos(\omega_1 2N\Delta t) \\ \mathrm{F} \\ \sin(\omega_m 2N\Delta t) \\ \cos(\omega_m 2N\Delta t) \end{array}\right\}. \tag{10}$$

In symbolic form, we can write

$$\underset{(Lp\times 2m)}{[\mathbf{A}]}\ \underset{(2m\times 1)}{\{\mathbf{b}_1\}} + \underset{(Lp\times 2N-2m)}{[\mathbf{C}]}\ \underset{(2N-2m\times 1)}{\{\mathbf{b}_2\}} = \underset{(Lp\times 1)}{\{\mathbf{E}\}} \tag{11}$$

and

$$\underset{(2m\times 2m)}{[\mathbf{B}]}\ \underset{(2m\times 1)}{\{\mathbf{b}_1\}} + \underset{(2m\times 2N-2m)}{[\mathbf{D}]}\ \underset{(2N-2m\times 1)}{\{\mathbf{b}_2\}} = \underset{(2m\times 1)}{\{\mathbf{F}\}}\ . \tag{12}$$

From Eq. (12),

$$\{\mathbf{b}_1\} = [\mathbf{B}]^{-1}[\{\mathbf{F}\} - [\mathbf{D}]\{\mathbf{b}_2\}] \tag{13}$$

and substituting in Eq. (11), we obtain

$$[[\mathbf{C}] - [\mathbf{A}][\mathbf{B}]^{-1}[\mathbf{D}]]\{\mathbf{b}_2\} = \{\mathbf{E}\} - [\mathbf{A}][\mathbf{B}]^{-1}\{\mathbf{F}\}. \tag{14}$$

From the over-determined system (14), $\{\mathbf{b}_2\}$ can be found as a least-square solution. $\{\mathbf{b}_1\}$ is then retrieved from Eq. (13). Together $\{\mathbf{b}_1\}$ and $\{\mathbf{b}_2\}$ provide the coefficients of Prony's polynomial. The roots of the polynomial will include the harmonic frequencies since the procedure presented here enforces it exactly.

3. Experimental example

In order to investigate the efficiency of the proposed procedure, experiments have been done for a pinned–pinned beam depicted in Fig. 1. A shaker was used to provide stationary noise excitation as well as harmonic loads. A total of 8192 discrete time samples have been obtained for each experiment. The data were sampled at 128 Hz. A preliminary analysis was carried out to know the mode shapes and the frequencies. Accordingly, we placed four accelerometers to ensure good level of measured response signal for the second bending mode. Harmonics were introduced close to the second bending mode frequency.

In the examples, modal parameters will first be computed only with stationary white-noise input to get the modal parameters as would be done in a normal OMA procedure. Then, we will consider the cases when harmonic excitations are present along with the random loads. Modal parameters are identified both with the polyreference LSCE method and by the single-reference LSCE modified to explicitly account for harmonic components (see previous section). When the polyreference LSCE method is used, correlation functions are computed with reference to the first two accelerometer signals, leading to eight sets of correlation functions in total. When the single-reference LSCE method is employed, we will consider correlation functions with reference to the first accelerometer only, leading to four sets of correlation functions. For both identification methods, the number of rows L in the data matrix is taken as $L = 200$ so that the total number of discrete correlation values used in the identification is $200 + 2N$ for each correlation function, N being the order of the identified model.

3.1. Pure stationary white-noise excitation

First the experiment was done with stationary white noise introduced between 18 and 28 Hz, to excite the second bending mode properly. Eigenfrequencies and damping values were identified with the polyreference LSCE method.

In Fig. 2 and Table 1, we report the convergence of the poles to the identified first, second and third eigenfrequencies when the order N of the system is increased.

It can be observed that the first three eigenfrequencies are identified. Note however that, since the stationary white-noise input was set for the experiment between 18 and 28 Hz, only the second eigenfrequency can be assumed to be identified properly by the OMA procedure. The second eigenfrequency was found to be 23.15 Hz and its associated damping is taken as 0.26%.

3.2. Single sine harmonic at 23.225 Hz

Let us now consider the case when a single-harmonic frequency is added to the response signal close to the second bending frequency. This is achieved by adding a sinusoidal harmonic

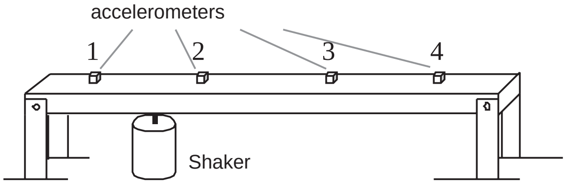

Fig. 1. Experimental set-up.

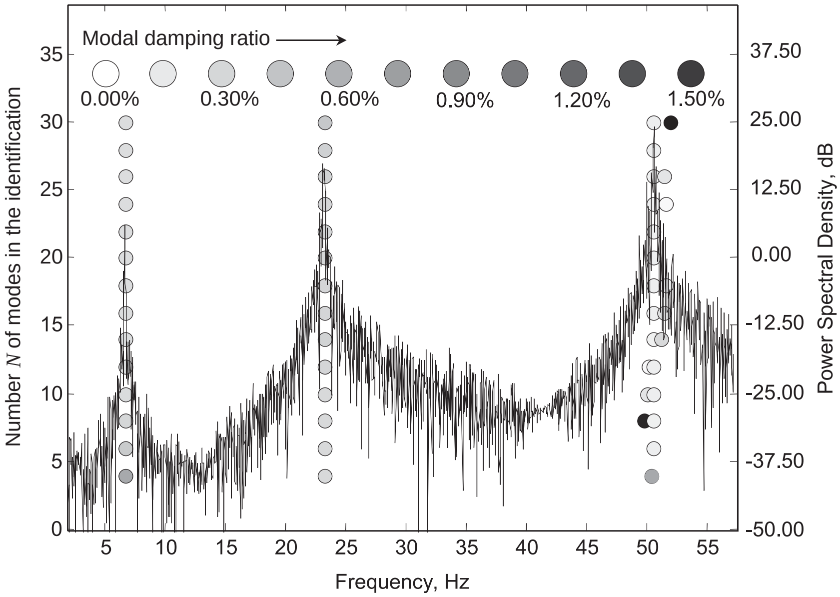

Fig. 2. PSD of acceleration at position 1 for pure random excitation and stability diagram for the polyreference LSCE method.

Table 1
Frequencies and associated damping (white noise only)

Modes	Polyreference LSCE					
	Freq.1 (Hz)	Damp.1 (%)	Freq.2 (Hz)	Damp.2 (%)	Freq.3 (Hz)	Damp.3 (%)
30	6.59	0.23	23.16	0.27	50.57	0.09
28	6.59	0.23	23.16	0.29	50.57	0.09
26	6.59	0.23	23.16	0.30	50.57	0.09
24	6.59	0.23	23.17	0.28	50.57	0.09
22	6.59	0.22	23.15	0.26	50.57	0.09
20	6.59	0.22	23.15	0.26	50.57	0.09
18	6.59	0.22	23.15	0.26	50.58	0.09
16	6.60	0.23	23.15	0.26	50.58	0.08
14	6.60	0.23	23.15	0.26	50.57	0.07
12	6.60	0.23	23.15	0.26	50.57	0.07
10	6.61	0.24	23.15	0.24	50.57	0.07
8	6.61	0.25	23.16	0.21	50.54	0.09
6	6.61	0.40	23.18	0.21	50.52	0.17
4	6.61	0.60	23.23	0.32	50.37	0.61

frequency to the input of the shaker along with the stationary white noise. As discussed in the theory, we have introduced two more equations in the Hankel matrix of the single-reference LSCE method to take into account the harmonic frequency.

In Fig. 3, the harmonic frequency at 23.225 Hz is clearly visible in the PSD plot because of its high peak. There is also a smaller peak at a frequency slightly below the harmonic frequency. From the PSD, it is not clear if that secondary peak corresponds to the second bending mode frequency. Applying the polyreference LSCE, the stabilization diagram identifies the dominating harmonic peak with an associated damping of the order of 0.05%. But the polyreference LSCE fails to identify the actual eigenfrequency properly even for an identification order of 40. Fig. 3 and Table 2 indicate a clear stable frequency line at 23.21 Hz computed by the Polyreference LSCE method. Remembering that the eigenfrequency is at 23.15 Hz for the second bending mode and that the harmonic frequency is at 23.225 Hz, one concludes that the polyreference LSCE identification procedure is strongly influenced by the presence of the harmonic component.

Fig. 4 and Table 2 report the results obtained when applying the modified single-reference LSCE approach proposed in this work. Obviously, since the harmonic component is explicitly introduced in the identification procedure, the harmonic has an associated zero damping. The identified eigenfrequency and its damping is very close to the second bending mode frequency and damping. Both frequency and damping values are nearly invariant at different modal order, which can be seen from Table 1 and Fig. 4. Hence, even for low modal orders, the modified single-reference LSCE can identify the eigenparameters accurately.

3.3. Two square-shaped harmonics at 23.30 and 23.80 Hz

In order to investigate the efficiency of the method when non-sinusoidal harmonics are present, we have introduced two periodic square wave excitations on top of the random loads, at frequencies 23.30 and 23.80 Hz. Note that the additional periodic loads have fundamental

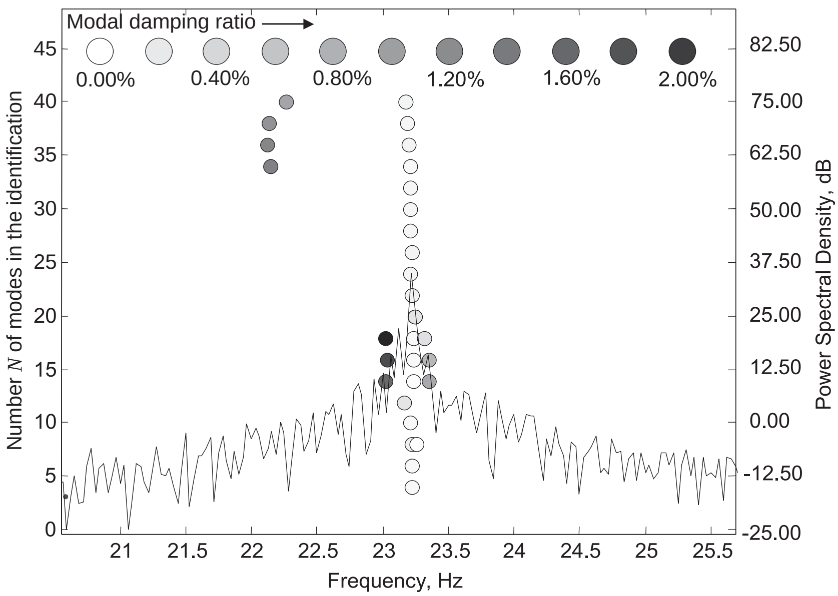

Fig. 3. PSD of acceleration at position 1 for a combined single-harmonic and random excitation, and stabilization diagram for the polyreference LSCE method.

Table 2
Frequencies and associated damping identified by the polyreference LSCE and the modified single-reference LSCE methods (random and single-harmonic excitation)

Modes	Polyreference LSCE				Modified Ref. LSCE	
	Freq.1 (Hz)	Damp.1 (%)	Freq.2 (Hz)	Damp.2 (%)	Freq.3 (Hz)	Damp.3 (%)
40	23.18	0.08	—	—	23.15	0.31
38	23.18	0.05	—	—	23.15	0.29
36	23.20	0.08	—	—	23.15	0.29
34	23.21	0.05	—	—	23.16	0.30
32	23.21	0.05	—	—	23.15	0.29
30	23.21	0.06	—	—	23.15	0.29
28	23.21	0.06	—	—	23.14	0.32
26	23.21	0.07	—	—	23.16	0.26
24	23.21	0.09	—	—	23.16	0.23
22	23.22	0.10	—	—	23.16	0.24
20	23.24	0.22	—	—	23.15	0.25
18	23.23	0.04	23.32	0.28	23.15	0.22
16	23.23	0.03	23.35	0.65	23.15	0.21
14	23.22	0.03	23.35	0.81	23.16	0.21
12	23.16	0.22	—	—	23.15	0.22
10	23.21	0.03	—	—	23.15	0.23
8	23.21	0.12	—	—	23.17	0.25
6	23.22	0.03	23.26	0.01	23.18	0.28
4	23.22	0.03	—	—	—	—

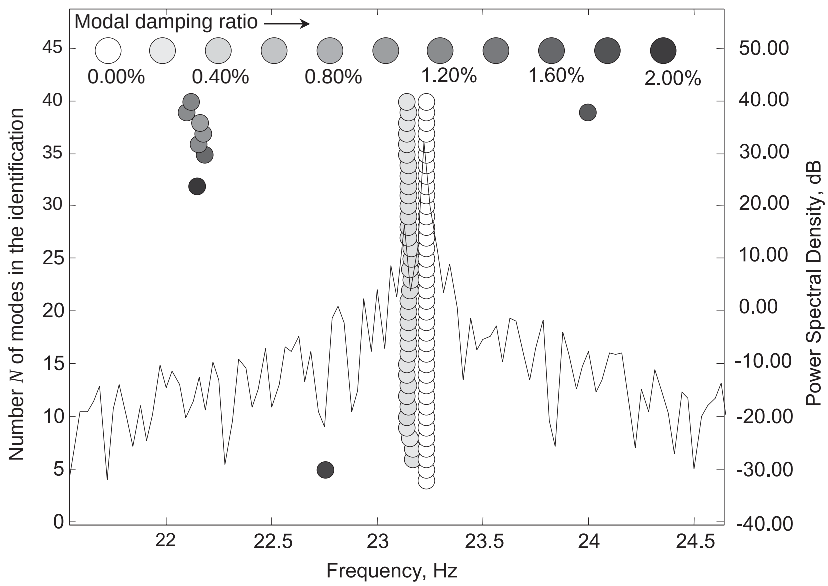

Fig. 4. PSD of acceleration at position 1 for a combined single-harmonic and random excitation, and stabilization diagram of the modified single-reference LSCE.

frequencies very close to the second bending mode frequency, namely 23.15 Hz. Also, because the periodic excitations have a square waveform, they have a very strong fundamental component so that, in the response, mainly the harmonic response corresponding to the fundamental frequencies will appear.

In Fig. 5, two peaks at 23.30 and 23.80 Hz are clearly observed. Again, a secondary peak is present slightly below 23.30 Hz, which might be due to the second bending mode frequency. From Table 3 and Fig. 5 it can be concluded that, when using the polyreference LSCE, the identified frequencies are associated to the two fundamental harmonic frequencies. Note that the identified damping corresponding to the harmonic frequencies is not small, while there should be null in theory. Hence, in practice, it would be difficult at this point to assimilate the identified modes to harmonic responses and the analyst would probably conclude that the identified parameters correspond to true eigenmodes.

When applying the modified single-reference LSCE, we have introduced in the Hankle matrix two harmonic components of frequencies corresponding to the periodic excitations, namely 23.30 and 23.80 Hz. The stabilization diagram is shown in Fig. 6. The last two columns in Table 3 give the identified frequencies and damping values. The identified harmonic parameters are not reported in the table since, by construction, they have the exact harmonic frequency and zero damping. It is seen that the identified eigenfrequency and the associated damping are very close to the expected values for the second bending mode. Therefore, whereas we failed to get any correct modal parameters from the polyreference LSCE method, the modified single-reference approach allows identification of the eigenparameters in a robust manner.

This example indicates that the modified single-reference LSCE method can be expected to be efficient for many general periodic loads that appear additionally to the random excitation.

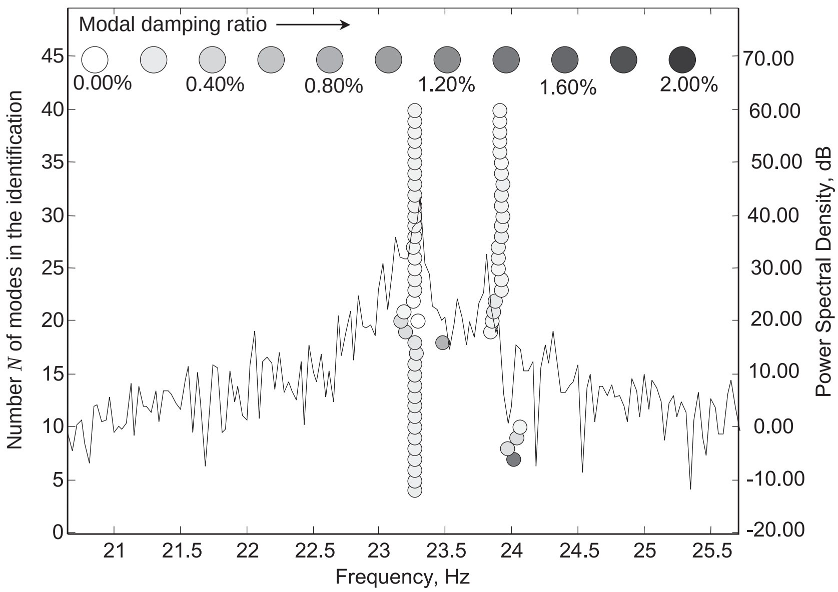

Fig. 5. PSD of the acceleration at position 1 for random loads and two additional periodic square excitations, and stabilization diagram from the polyreference LSCE method.

Table 3
Frequencies and associated damping for both polyreference LSCE and single-reference LSCE method (two additional periodic excitations with square waveform)

Modes	Polyreference LSCE				Modified LSCE	
	Freq.1 (Hz)	Damp.1 (%)	Freq.2 (Hz)	Damp.2 (%)	Freq.3 (Hz)	Damp.3 (%)
40	23.26	0.11	23.90	0.07	23.17	0.23
39	23.26	0.11	23.90	0.03	23.17	0.24
37	23.26	0.11	23.90	0.04	23.17	0.25
35	23.26	0.12	23.91	0.04	23.17	0.25
33	23.26	0.12	23.93	0.14	23.17	0.25
31	23.26	0.11	23.92	0.10	23.17	0.26
29	23.26	0.11	23.92	0.09	23.17	0.28
27	23.25	0.08	23.89	0.12	23.18	0.30
25	23.27	0.04	23.89	0.06	23.17	0.27
24	23.26	0.09	23.92	0.11	23.17	0.25
23	23.26	0.10	23.92	0.11	23.18	0.27
22	23.25	0.02	23.87	0.19	23.18	0.27
20	23.29	0.01	23.84	0.09	23.19	0.29
18	23.27	0.24	23.47	0.70	23.19	0.34
16	23.27	0.16	—	—	23.19	0.31
14	23.27	0.16	—	—	23.18	0.37
12	23.26	0.16	—	—	23.16	0.43
10	23.26	0.15	24.05	0.07	23.14	0.26
9	23.26	0.15	24.03	0.29	23.13	0.28
8	23.26	0.14	23.96	0.28	23.34	0.79
7	23.26	0.14	24.01	1.26	—	—

Indeed, if the periodic excitation has a waveform such that significant superharmonic components are present, one can easily introduce in the modified single-reference LSCE several superharmonics for the identification.

3.4. Three additional harmonics at 22.40, 22.65 and 22.90 Hz

Now three sine harmonic frequencies at 22.40, 22.65 and 22.9 Hz are given as input to the shaker along with the stationary white noise in order to investigate the effectiveness of the new method in the presence of strong multi-harmonics. As mentioned in the theory, we have introduced all three harmonics in the Hankel matrix of the single-reference LSCE method to take into account the harmonic frequencies.

In Fig. 7, three harmonic frequencies can be clearly identified at 22.40, 22.65 and 22.90 Hz. We also observe just after the third harmonic frequency, a peak which corresponds to the second bending mode frequency.

As shown in Table 4 and Fig. 7, the polyreference LSCE method exhibits two stabilization lines. Those lines are at 22.47 and 22.90 Hz, which correspond to two of the harmonic frequencies introduced in the signal. There is no sign of the second harmonic frequency in the graph.

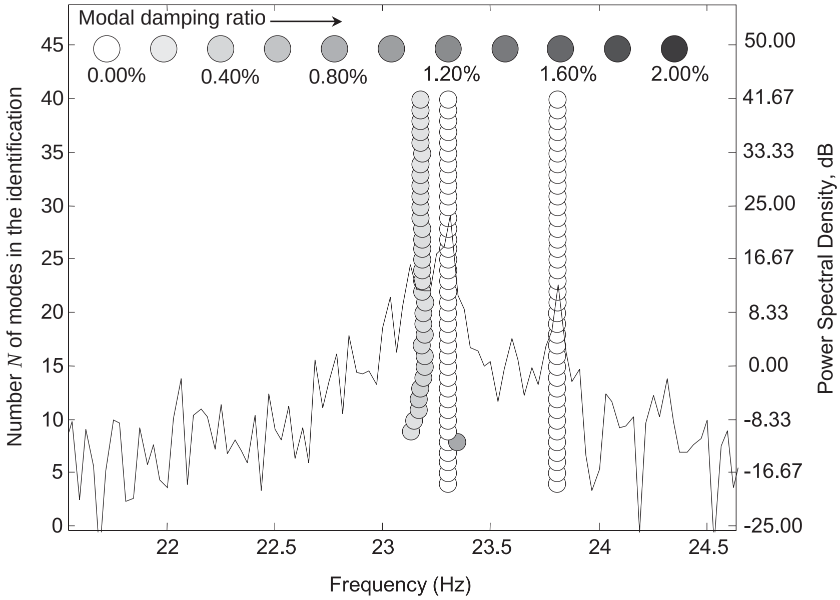

Fig. 6. PSD of the acceleration at position 1 for random loads and two additional periodic square excitations, and stabilization diagram from the modified single-reference LSCE method.

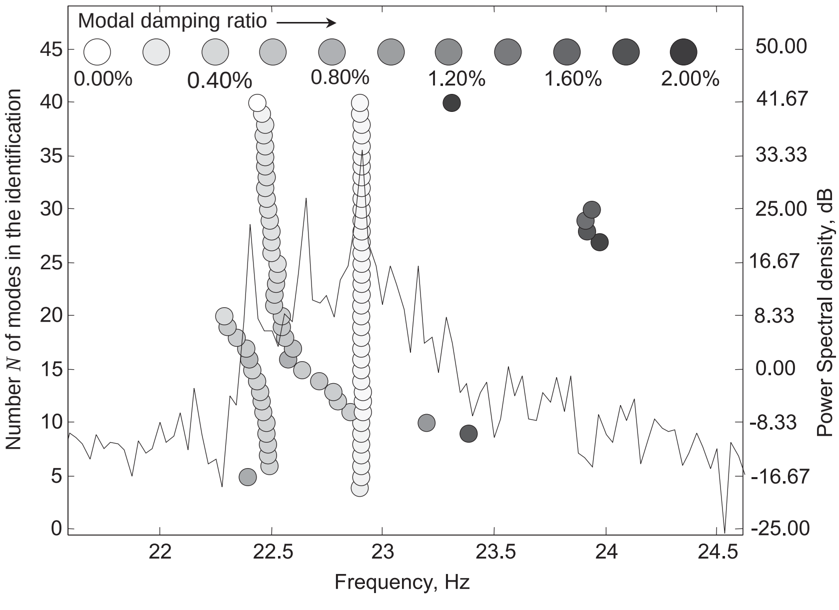

Fig. 7. PSD of the acceleration at position 1 for random loads and three additional harmonic excitations, and stabilization diagram from the polyreference LSCE method.

Although in theory those frequencies should be associated to zero damping, the results in Table 4 indicate that the damping of the identified harmonic parts becomes small only for high modal orders. The polyreference LSCE does not identify the actual eigenfrequency.

Table 4
Frequencies and associated damping from the polyreference LSCE and the modified single-reference LSCE methods (three additional harmonic excitations)

Models	Polyreference LSCE				Modified LSCE	
	Freq.1 (Hz)	Damp.1 (%)	Freq.2 (Hz)	Damp.2 (%)	Freq.3 (Hz)	Damp.3 (%)
40	22.44	0.00	22.90	0.04	23.13	0.18
39	22.46	0.10	22.90	0.06	23.13	0.17
38	22.47	0.19	22.90	0.06	23.13	0.17
37	22.46	0.23	22.90	0.07	23.13	0.20
36	22.47	0.23	22.90	0.06	23.14	0.20
35	22.47	0.25	22.90	0.06	23.13	0.25
34	22.47	0.29	22.90	0.07	23.14	0.27
33	22.47	0.31	22.90	0.07	23.14	0.31
32	22.47	0.27	22.90	0.07	23.13	0.31
31	22.48	0.26	22.90	0.06	23.12	0.36
30	22.48	0.27	22.90	0.06	23.14	0.50
29	22.49	0.26	22.90	0.06	23.13	0.54
28	22.49	0.27	22.90	0.06	23.13	0.54
27	22.49	0.28	22.90	0.06	23.15	0.54
26	22.50	0.26	22.90	0.06	23.15	0.52
24	22.52	0.35	22.90	0.06	23.18	0.50
22	22.51	0.40	22.90	0.06	23.17	0.70
20	22.54	0.43	22.90	0.06	23.18	1.50
18	22.54	0.49	22.90	0.06	—	—
16	22.57	0.72	22.90	0.06	—	—
14	22.43	0.40	22.90	0.05	—	—
12	22.45	0.39	22.91	0.05	—	—

Fig. 8 and Table 4 represent the stability diagram for the modified LSCE method proposed in this paper. The first three frequency lines belong to the three harmonics, and their frequencies have been set so as to exactly match the harmonic frequencies, the associated damping being zero (those frequencies and damping values are not given in the table). The other identified frequency and damping is listed in Table 4 and can be seen to be very close to the frequency and damping of the second mode identified earlier. Note however that the damping ratio is slightly underestimated.

3.5. Robustness of the modified LSCE

From the experiments described above, it can be concluded that the polyreference LSCE method cannot identify the eigenfrequencies accurately when harmonic frequencies close to the natural frequencies are present in the signal. However, the modified single-reference LSCE approach proposed here allows identification of the modal parameters with high confidence in that case.

In order to further illustrate the robustness of the new approach and to investigate its sensitivity to the interval between eigenfrequency and harmonic frequency, we show in Fig. 9 the computed

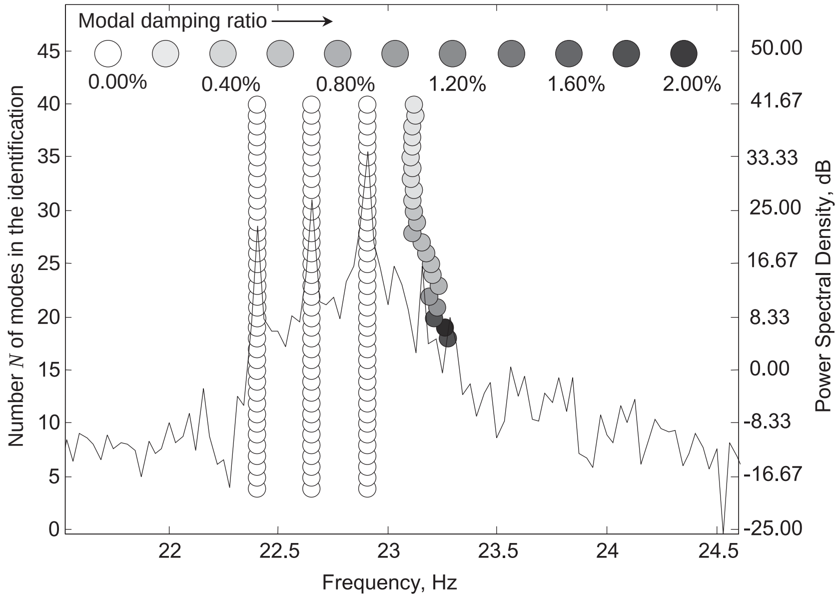

Fig. 8. PSD of the acceleration at position 1 for random loads and three additional harmonic excitations, and stabilization diagram from the modified single-reference LSCE method.

modal parameters as a function of the harmonic frequency. We recall that the eigenfrequency corresponding to the second bending mode is 23.15 Hz. In this example, the level of the harmonic input relative to the level of random excitation was kept constant.

Fig. 9 shows that the modified single-reference LSCE can identify the eigenfrequency with an accuracy of about 0.01 Hz for a harmonic component 0.2% lower or 0.4% higher than the eigenfrequency. It also identifies the damping ratio within 0.05% for a harmonic frequency 0.5% lower or 0.3% higher than the eigenfrequency. The polyreference method fails to identify the eigenfrequency and the corresponding damping accurately when the harmonic frequency is so close to the eigenfrequency. Hence, clearly, the modified single-reference LSCE significantly improves the robustness of the identification when the harmonic frequency is close to an eigenfrequency.

Obviously, in the modified LSCE method proposed here, it is important to introduce the exact harmonic frequency in the identification a priori. In Fig. 10, we consider the case where the harmonic frequency specified in the identification with the modified LSCE algorithm is different from the exact harmonic frequency, namely 23.225 Hz in this case. One observes that if a harmonic frequency different from the actual 23.225 Hz of the input excitation is specified, the identified frequencies vary linearly with respect to the harmonic frequency input in the vicinity of the exact harmonic frequency. When the harmonic frequency used in the identification is significantly higher than the exact harmonic frequency, the identified eigenfrequency converges to the one identified by the standard polyreference LSCE method whereas the identified damping ratio decreases significantly. Hence, it is essential to introduce in the modified LSCE algorithm a harmonic frequency very close to the actual one. Otherwise, the harmonic component explicitly

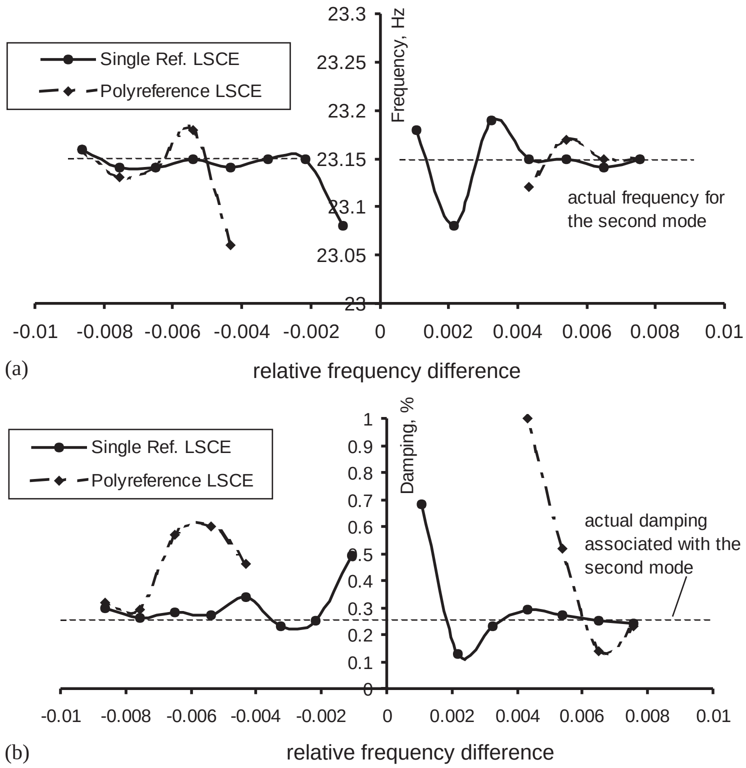

Fig. 9. Identified eigenfrequency (a) and damping ratio (b) for the second bending mode as a function of $(\omega_{\text{harmonic}} - \omega_{\text{bending}})/\omega_{\text{bending}}$.

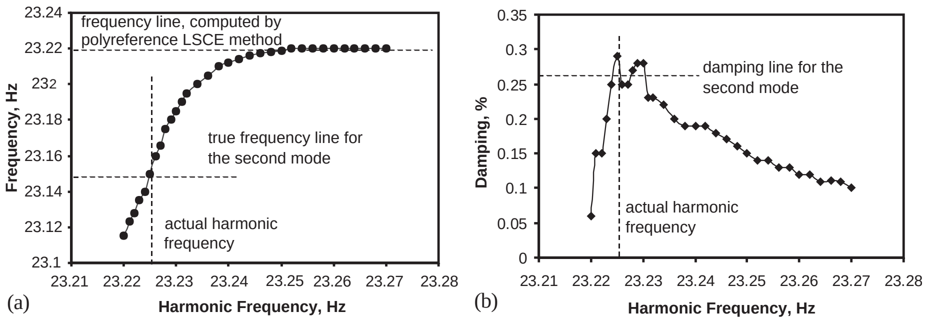

Fig. 10. Identified modal frequency (a) and damping ratio (b) as a function of the harmonic frequency specified in the modified LSCE.

included in the identification is not capable of representing the harmonic part of the signal properly and the modification of the LSCE gives no benefit. In the worse case, the modification in the LSCE has no effect and the identified parameters are the ones obtained by standard LSCE algorithms.

4. Possible further developments

The method proposed in this work was found to be efficient in identifying modal parameters in the presence of harmonic excitation with a frequency close to an eigenfrequency. It is therefore an interesting method for applications where harmonic loads are inherently present together with operational noise excitation. For instance, we are planning to apply the method to machines with rotating components such as tire test rigs and wind turbines. It can easily be extended to include any general periodic component by considering its Fourier expansion.

The modified LSCE procedure can also be extended to explicitly include damped oscillations known a priori. This could be helpful in order to allow the identification of modes only weakly present in a signal and which are masked by eigenmodes strongly dominant. The procedure would then first consist of identifying the dominant components and then, in a second step, include the dominant modes in the LSCE explicitly to identify the modes weakly present in the signal. Other research directions currently considered also include the application of the proposed modified LSCE to the identification of super- and sub-harmonics in non-linear systems.

5. Conclusion

We have proposed a modification of the single-reference least-square complex exponential (LSCE) identification method used in the context of operational modal analysis. This modification allows one to explicitly account for harmonic contents in the measured signal.

When the harmonic excitation part has a frequency close to eigenfrequencies that one tries to identify, standard LSCE methods fail to identify the modal parameters accurately. In that very challenging situation, the modified LSCE proposed here yields accurate identification even if the harmonic excitation frequency is very close to an eigenfrequency. Even in the case where the standard polyreference LSCE can identify the modal parameters accurately (that is when the harmonic frequency is not too close to an eigenfrequency), the proposed modified LSCE algorithm yields accurate parameters for a much lower assumed system order.

The experimental example described here illustrates the efficiency and robustness of the modified LSCE. It is also shown that, when the harmonic frequency input in the modified LSCE is different from the actual harmonic frequency in the signal, the modified LSCE simply degenerates to a standard algorithm.

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